A Guide to the math component of the Digital SAT

Topics

I. The Objective of this Book

In any high stakes test (especially the digital SAT) There are three critical components that the student needs to master:

- Subject Knowledge
- Time Management
- Carelessness

We have created an online practice application: https://www.satdiagnostics.com that addresses the three components and is closely linked to this book. It is private and anonymous. There is no user id, password or tracking.

The primary goal is to provide a plan of attack. If you have seen and practiced enough problems, then in the first few seconds you can formulate:

- The concept behind the problem
- The first few steps leading to a solution
- Misdirection and pitfalls (words or logic)

We start by examining the possible algorithms that describe how the College Board generates module I and module II-easy or module II-hard questions (weights and distribution) and the effect on a student's overall math score

This book is the culmination of years of working with some of the brightest and most motivated students in America.

Thank you in advance for purchasing and more important, using this book

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II. Resources

The list below covers our recommended resources:

I. https://satsuitequestionbank.collegeboard.org/ - This is a downloadable set of paper-based questions from previous SAT tests that are selectable by topic and difficulty (there are 343 difficult questions).

II. https://bluebook.collegeboard.org/students - This is the entre into College Boards six simulated online tests.

III. https://www.khanacademy.org/ - Khan academy is a free site with close ties to the College Board. Also look into their trial 'boot camp'.

IV. https://testinnovators.com/ They charge approximately $200 for a complete package (verbal and math). The math problems closely emulate the College Board blue book and there are 10 simulated tests. The explanations are concise and very clear. In addition, they have many supporting videos.

Our recommendation is to first complete items IV and some of the more difficult problems from I

Then work on II and III. The results of each Blue Book test directs you to appropriate problems in Khan.

III. The scoring algorithm behind the Digital SAT

The 2024 Online Digital SAT incorporates a two-stage hybrid adaptive test format. This means that the score on the first module (mod1) of the Math component dictate whether the second mod the student gets is easy or hard. Each mod contains 22 questions (only 20 are scored – the other two are experimental to help design future tests). ***The questions themselves are NOT adaptive BUT they are WEIGHTED.***

This version dramatically affects students who are either: (1) not prepared for the most difficult questions or (2) tend to be careless (due more to misreading then calculation). If the student makes it to mod2-hard, they may likely score between 700 – 800. Alternatively, mod2-easy may limit them to the 500 – 600 range. This is all due to the weighting algorithms.

In essence, if you don't move to mod2-hard your grade could result in a 200-point difference. This is a direct result of the combination of question weights and frequency of hard questions in mod2 hard versus easy.

A Plausible Explanation

College board not only does not publish their algorithms or weights (of questions). They may also change the weights and their distribution as data from previous test results become available. Therefore, we will present a hypothetical model which helps explain the importance of understanding the theory behind the algorithm.

Assumptions:

- Question Weights:
  - Difficult = 1.2

- Medium = 1.0
- Easy = 0.8

- Distribution of weighted questions mod1, mod2H and mod2E
  - Mod1: H = 7, M = 7, E = 6
  - Mod2.H: H = 12, M = 5, E = 3
  - Mod2.E: H = 3, M = 10, E = 7
- Percent Correct
  - Student A: H = 0.8, M = 0.9, E = 1.0
  - Student B: H = 0.6, M = 0.8, E = 0.9

Results: As reported in the simulation below. With both weighting and non-weighting, the scores are similar, but student A's total score would be in the high 700's while student B would be in the low to mid 500's. A two-hundred-point difference.

Therefore, the **primary objective** of this book and the associated software applications is to increase the odds of a student moving from: **Module 1 to Module 2 Hard**

user inputs	factors	weights		mod 1 dist	mod 2 Hard	mod 2 easy			
Difficult	0.2	1.2		7	12	3		NON WEIGHTED	
Medium	1	1		7	5	10	raw score	H - range	E - range
Easy	0.2	0.8		6	3	7	26	580 - 640	430 - 490
factors are relative to medium							27	600 - 660	450 - 510
			% correct (as decimal)				28	610 - 670	460 - 520
			mod 1	mod 2	mod 2		29	630 - 690	470 - 530
	Group		difficult	medium	easy		30	640 - 700	490 - 550
	m2-H students		0.8	0.9	1		31	660 - 720	500 - 560
	m2-E students		0.6	0.8	0.9		32	680 - 740	520 - 580
							33	700 - 760	530 - 590
weighted			mod 1	mod 2	raw score	raw score	34	720 - 780	580 - 640
			raw scor	mod II Hard	Mod II Easy	tot mod I & II	35	740-800	540 - 620
	m2-H students		17.82	18.42	X	36.138	36	760 - 800	580 - 660
	m2-E students		14.96	X	15.2	30.1192	37	770 - 800	x
							38	780 - 800	x
non weighted			mod 1	mod 2	raw score	raw score	39	780 - 800	x
			raw scor	mod II Hard	Mod II Easy	tot mod I & II	40	780 - 800	x
	m2-H students		19.4	17.1	X	36.891			
	m2-E students		15.2	X	16.1	31.147			

It is critical that the student master the hard problems.

IV. The Trichotomy Principal

There three different cases to consider:

- Quadratic Equations with a variable constant or coefficient.
- System of Linear Equations.
- Comparison of two Linear Expressions.

And are three outcomes with this principal:

- Two or more solutions
- Exactly one solution
- No solutions

Case 1. Quadratic equation with different outcomes

Concept: Given a quadratic in standard form:

$$y = ax^2 + bx + c$$

This represents, geometrically, a parabola, which faces up if $a > 0$ (vertex = min value) or down if $a < 0$ (vertex = max value).

An alternative to factoring (if possible) is using the quadric solution:

$$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$$

Let D (the discriminant) = $b^2 - 4ac$.

- If $D > 0$ then the quadratic will intercept the x-axis at <u>two different points</u> giving two solutions $x = (–b \pm \sqrt{D})/2a$.
- If $D = 0$ then then the quadratic will intercept the x-axis at <u>a single point</u> giving a single solutions $x = –b/2a$ which is

referred to as the line of symmetry (the parabola is symmetry with respect to this line).

- If D < 0 then, since the square root cannot contain a negative number, there are no Real solutions. It's a perfectly good parabola, it just lies either totally above or below the x-axis and has complex (imaginary) solutions.

Example 1. Given f(x) = $2x^2 + 4x + d$. For what value of d, will f(x) have two distinct real solutions?

Concept: A quadratic equation has **two** real solutions when the discriminant > 0.

Solution: Discriminant = $b^2 - 4ac > 0$ Therefore 16 – 8d >, d < 2.

Example 2. Given f(x) = $2x^2 + 4x + d$. For what value of d, will f(x) have exactly one solution?

Concept: A quadratic equation has **one** real solution when the discriminant = 0.

Solution: The discriminant = $b^2 - 4ac = 0$ Therefore 16 – 8d = 0, d = 2.

Example 3. Given f(x) = $2x^2 + 4x + d$. For what value of d, will f(x) have no real solutions (just complex)?

Concept: A quadratic has no real solutions solutions when the discriminant < 0.

Solution: The discriminant = $b^2 - 4ac < 0$ Therefore 16 – 8d < 0, d > 2.

Case II. System of Linear Equations with Different Outcomes

The Concept: Given a System of linear equations in standard form:

$$ax + by = c$$

dx + ey = f

This represents, geometrically, a two lines in the x-y plane. There are three outcomes:

- The lines are parallel which implies that the slopes are equal, and the y-intercepts are different. Therefore, there are no solutions – the lines don't meet.
- The lines are collinear. The equations are identical by some multiple. Therefore, they have and infinite number of solutions
- The lines intersect. The slopes are different, and they have exactly one solution

To for a linear equation in standard the slope equals -a/b and the y-intercept is obtained by letting x = 0 and therefore y-intercept = c/b

The Concept: If you are given a linear system of equations with a coefficient or constant equal to a variable and asked whether it has 1, 2 or no real solutions **you should think slope and y-intercept.**

Example 4. Given ax + 4y = 10 and 8x + 16y = 30. For what value of a, will the system have no solution?

Solution: To have no solution the lines must be parallel. This implies that the slopes are equal, and the y-intercepts are different. Comparing the slopes: **-a/4 = -8/16 => a = 2.** Comparing the y-intercepts: **10/4 ≠ 40/16**

Example 5. Given ax + 4y = 10 and 8x + 16y = 40. For what value of a, will the system have an infinite number of solutions?

Solution: To have an infinite number of solutions the lines must be the colinear. This implies that both the slopes and y-intercepts are the same. Comparing the slopes: -a/4 = -8/16 => a = 2. Comparing the y-intercepts: 10/4 = 40/16. Another way to solve this is to show that one equation is a multiple of the other.

Example 6. Given ax + 4y = 10 and 8x + 16y = 40. For what value of a, will the system have exactly one solution?

Solution: To have an exactly one solution the lines must intersect. This implies that the slopes are different with no restrictions on the y-intercept. Comparing the slopes: -a/4 = -8/16 => a ≠ 2.

Case III. Comparison of two Linear Expressions with different outcomes

The Concept: Given two linear expressions: ax + b = cx + d. There are three possible outcomes:

- a = c and b = d. The expressions are the same and the equality will hold regardless of the value of x. Therefore, there are an **infinite number of solutions.**
- a = c and b ≠ c. The constants are different. Since b ≠ c there are **no values of x resulting in equal expressions**.
- a ≠ c. The coefficients of x are different and **there is a unique** solution: x = (d – b)/(a – c).

The Concept: If you are given two linear expressions with a coefficient or constant equal to a variable and asked whether there are 1, 2 or no real solutions **you should think whether the x coefficients are equal and whether the constants are equal**.

Example 7. Given ax + 5 = 3x + 5. For what value of a, will the system have infinitely many solutions.

Solution: if a = 5 then the expressions are the same.

Example 8. Given ax + 5 = 3x + 5. For what value of a, will the system have infinitely many solutions.

Solution: if a = 5 then the expressions are the same.

Example 9. Given ax + 5 = 3x + 5. For what value of a, will the system have infinitely many solutions.

Solution: if a = 5 then the expressions are the same.

V. Linear Equations and Systems

The Concept: There are two primary ways in which a linear equation can be expressed:

- Standard Form: $ax + by = c$
- Slope Intercept: $y = mx + b$, where m is the slope and b is the y-intercept

In addition, Unit measures: miles/gal or gal/miles are equivalent to slopes. Miles/gallon measures how many miles we can get on just one gallon of gasoline, while gallons/miles measure how many gallons we need to travel just one mile.

An Example of standard form:

- There are x small bottles each containing a fluid ounces and y large bottle each containing b fluid ounces the combined number of fluid ounces of both bottles is c.

An Example of Slope Intercept:

- A plumber charges a flat fee of $b (just for coming out) and hourly fee of $m.

The Concept: Determine the type of linear and if a rate is to be considered, for the unit variable determine what is the numerator vs the denominator.

Example 10: **The grocery carries two types of fruits Apples (A) at $5 per pound and Peaches (P) at $3 per pound. In total 50 pounds of fruit were sold**

- **What is the maximum and minimum totals sales?**
- **If 10 pounds more apples were sold then peaches, what would be the total received?**

Solution: **For the first part the equation (in standard form):**

$$T = 5A + 3P$$

- **The max total occurs when we sell 50 pounds of just apples = $250. The min total occurs when we sell 50 pounds or just peaches = $150**
- **The equivalent algebraic statement: A = P + 10. Therefore, A + P = 50 => P + 10 + P = 50 implies that 2P = 40 and P = 20. Therefore ,the total = $5*30 + $3*20 = $150 + $60 = $210**

Example 11: Given a linear equation f(x) such that f(2) = 5 and f(7) = 20. If another linear equation g(x) is perpendicular to f(x) and g(3) = 12, what is g(12)?

Solution: The approach will be to first find the slope of f(x) and then find the slope of g(x) which is the regative reciprocal (a property of perpendicularity). Finally we plug the given point on g(x) to get it's linear equation.

- The slope of f(x) = [f(7) – f(2)]/[7 – 2] = [20 – 5]/5 = 15/5 = 3. The slope of g(x) = –1/3.
- Therefore g(x) = –(1/3)x + b, where b is the y–intercept.
- To determine b, since g(3) = 12, plug in the point (3,12) into g(x) = –(1/3)x + b
- 12 = –(1/3)(3) + b, which implies that b = 13
- Therefore, g(x) = –(1/3)x + 13
- And g(12) = –(1/3)12 + 13 = 9.

Example 12: A taxi companies cab travel k miles per week. It gets j miles per gallon of gas. Gas cost $n per gallon. If the company wishes to save $p dollars, what formula could be used to find p in terms of the other variables.

Solution: To turn a problem from theoretical (with only variables) to a more intuitive one, plug in numbers for the variables.

- First determine how many gallons (or fractional part) is required to travel 1 mile = 1/j (if you get 5 mpg then you can travel 1 mile on 1/5 of a gallon).
- Since gas cost \$n per gallon, then the cost to travel one mile = \$n(1/j) (suppose n = \$10, then the cost = \$10(1/5) = \$2 to travel 1 mile).
- Finally, to save \$p, let x = the number of less miles they need to drive. Therefore: x = p/(n/j) = pj/n is the number of less miles required to \$p. Suppose that p = \$40. According to our scenario it cost 10/5 (\$2) to travel 1 mile then: x = 40/(10/5) = (40/2) = 20 less miles.

Example 13: The cost to enter an amusement park is composed of a onetime fee of \$70 plus a per person fee of \$20 for the first 15 people in the group and an additional \$10 for each additional person. if n > 15 people are in the group, what is linear equation c(n) that give the total cost for admission for this group?

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Solution: Start by a literal translation:

- **$C(n) = 70 + 20*15 + 10()(n - 15)$**
- **$C(n) = 370 + 100n - 1500$**
- **$C(n) = 100n - 1130$**

Example 14: $3x + 7y = 12$ is translated down 5 units. What are the coordinates of the x and y intercepts of the new equation

Solution:

Example 14: An individual has $300 available to purchase either blue pens at $1 each or red pens at $2 each. What is the maximum number of red pens they can purchase?

Example 15: Given a triangle with two sides: 8 and 13, what is the minimum and maximum that the third could be?

The Concept: The Triangle inequality states that given three sides of triangle: a, b and c, any side c must be strictly less than the sum of the other two sides and strictly greater than their difference: $|b - c| < a < b + c$. Note that we use an absolute value on the left side, since with variables we don't know which is greater but the difference must be positive.

Example 16: Tom gets $7 per hour for the first 10 hours and $10 per hour after that in a given week. If tom saves 70 % or his wages, what is the min number of hours he must work to save $200.

VI. Exponents and Quadratics

Concepts: quadratic equations can take on many forms.

1) $f(x) = ax^2 + bx + c$ ... standard form
2) $f(x) = a(x - h)^2 + k$ … vertex form, where (h, k) is the vertex
3) $f(x) = a(x - s1)(x - s2)$ ... factored form, where s1 and s2 are x-intercepts

In all cases, if $a > 0$ the parabola faces up – a minimum point, otherwise down – a maximum. One of the key coordinates of a quadratic equation is vertex, which is either a min or max point. The x-coordinate of the vertex can be derived from the line of symmetry, for the standard, $x = -b/2a$ or the average of the x-intercepts. The y-coordinate can be evaluated by replacing x in f(x) with the x-coordinate of the vertex.

For practice and additional problems go to satdiagnostics.com and select the quadratics subject.

Example 17: **Given the quadratic** $y = x^2 + 2x - 15$. What is it's minium or maximum value and where does it occur?

The Concept: If we consider the solution to a quadratic in standard form: $y = Ax^2 + Bx + C$. Note that geometrically, a quadratic is a parabola.

$$y = [-B \pm \sqrt{(B^2 - 4AC)}]/2A$$

Then the line of symmetry: $x = -B/2A$. The x value is also the x value of the vertex. If the coefficient A is positive then the parabola faces upward and has a minimum, otherwise the reverse. This x coordinate specifies WHERE the minimum occurs. To find what the value is (the y-coordinate of the vertex), plug the x-value into the quadratic equation.

Solution:

Example 18. Given the quadratic $y = x^2 + 2x - 15$, what are the x-intercepts (solutions), What is its min or max?

Solution:

Example 19. Given the quadratic $y = (x - 3)(x + 5)$. Where does the minimum occur what is its value?

Solution:

Example 20. Given the exponential equation:

$$f(t) = a(0.35)^t$$

where t represents the time-period in months. If $f(0) = 300$, what is $f(2)$.

Concept: In the exponential equation $f(t) = a(1 \pm b)^t$:

- a represents the initial value when $t = 0$.
- b represents the rate. $1 \pm b$ represents 100% of the prior iteration $\pm b$

- If $b>0$ then the this is a growth function. Otherwise, $b<0$ we have a :

Solution:

Example 21. Given the exponential equation:

$f(t) = a(b)^t$. If $f(0) = 300$ and $f(1) = 600$, then what does $f(2)$ equal?

Example 22: $f(x) = (x - 5)(x + 3)(x - 2)$. If $g(x) = f(x) - 3$ and $h(x) = f(x - 2)$, what is the value of $f(0) + g(0) + h(0)$?

Example 23: Given the formula for the height of a projectile: $h(t) = 16t - 32t^2 + 20$. What is its height. At what time does it reach It and when does it hit the ground?

Example 24: **F(t) = 1000(0.25)^(t/6) describes the depreciated value of a car after t months. If the value decreases each year by p% of its value from the previous month, what is the value of p?**

Example 25: Given the quadratic equation:

$$f(x) = x^2 - 6x + 15$$

If a = sum of the x-intercepts and b = the product, what is the value of a+b

Concept: If r1 and r2 are the x-intercepts (the solutions) of the quadratic equation $f(x) = x^2 + bx + c$, then **$f(x) = (x - r1)(x - r2)$ and therefore $f(x) = x^2 - (r1 + r2)x + r1r2$.** Therefore, b = **– (r1 + r2) and c = r1r2**

Solution: **a = –(–6) = 6** and b = 15. Therefore, a + b = 21

Example 26: If $f(x) = 3(x-2)^2 + 5$ is the vertex form of the quadratic equation. And $f(x) = ax^2 - bx + c$ is its standard form, what are the values of a, b and c?

Example 27: $f(x) = 8(3)^x$. $g(x) = f(x+3)$. What does $g(2)$ equal?

Solution: $g(2) = f(2+3) = f(5) = 8(3)^5 = 8 * 243 = 1{,}944$

Example 28: If $f(x) = a/(x + b)$ and $g(x) = f(x+5)$, and $h(x) = f(x) + 5$. What are the vertical asymptotes of $f(x)$, $g(x)$ and $h(x)$?

Example 29: **What is the solution(s) to the equation:**

$$\sqrt{x^2} = 4x - 3$$

Concept: $\sqrt{x^2} \neq x$ because if $x < 0$ then the left side is positive while the right side is negative

Solution: $\sqrt{x^2} = 4x - 3 \Rightarrow |x| = 4x - 3$. Therefore, either

$X = 4x - 3$ or $x = 3 - 4x$. So $x = 1$ or $3/5$

Example 30 Solve: $(8x^2)/(x^2-16) - (4x)/(x + 4) = 2/(x - 4)$

Example 31 The following equation is true for all values of

X: $(ax + 5)(4x^3 - bx + 3) = 15x^3 + 12x^2 - 7x + 20$.

What is the value of $a + b$?

Example 32. Given the equation:

$(\sqrt{x^3})/(\sqrt[3]{x^2})$ = x raised to the fractional power a/b

What is a/b?

Example 33. Given the equation:

$3/(x - 4) + 2/(x - 3) = (ax + b)/(x - 4)(x - 2)$

Is true for all values of $x > 4$, what is the value of ab?

Example 34. Simplify $\sqrt[3]{(64k^3)}(\sqrt{(64k)})^2$

Example 35: the ratio of a to b is 3:7 and the ratio of b to c is 14:20. What is the ratio of a to c?

Example 36: A is 60% less then B which in turn is 40% greater than C. What is the percent of A to C?

Example 37. The population of a town is 15,000. A random sample of 500 is taken. From this sample the probability of than a person is less than 30 years old is 20% with a margin of error of 0.5%. What is the least number of people in the general population that could be over 30 years old?

Example 38. Tom mixed a batch of paint using 3 pounds of red paint with 7 pounds of green paint. Paul must create a similar batch using the same ratio of paint as Tom. If Paul uses 21 ounces of red paint, how much green paint should he use?

Example 39. Group A has 8 people with an average weight of 140 pounds. Group B has 6 people with an average of 110 pounds. If Groups A and B are combined, what is the average weight?

VII. Geometry with Diagrams

Example 40.